3. V. M. Voloshchuk and Yu. S. Sedunov, Coagulation Processes in Dispersed Systems [in Russian], Leningrad (1975).
4. A. S. Monin (ed.), Diffusion and Contaminants of Air [in Russian], Moscow (1962).

STABILITY OF THE LAMINAR BOUNDARY LAYER WITH STRONG BLOWING

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UDC 532.526

The authors present a method of calculating nonsimilarity laminar boundary layer flow over a permeable flat plate with uniform blowing.

As was shown in [1, 2], the boundary-layer equations for zero-gradient flow over a permeable flat plate can be solved for a finite range of variation of the blowing parameter F_w. As F_w tends to a critical value the boundary layer thickness increases without bound, while the friction coefficient tends to zero. For the case of uniform blowing along the length of the plate similar conclusions were reached in [3, 4], from direct numerical solution of the boundary layer equations. However, the results of the experimental investigation of [5] indicate that the laminar flow regime can exist for large enough intensities of transverse mass flow. It was shown in [5, 6] that as the blowing parameter increases there is a gradual deformation of the velocity profile from the Blasius to a sharply pronounced S-shape typical of jet type flows. When F_w reaches the critical value one does not observe a sharp increase of the boundary layer thickness nor a change of the flow regime, i.e., the boundary layer separates smoothly from the wall. A simple analytical solution of [7] gives good agreement with the experimental data at moderate blowing intensities, as was shown in [6]. The unsatisfactory agreement between the theoretical and experimental velocity distributions with strong blowing is due primarily to the negative pressure gradient induced by the transverse mass flow, which is not accounted for in either the numerical solutions [3, 4] or the analytical solution [7].

1. To establish (i.e., find) the velocity distribution in the unperturbed boundary layer with uniform blowing, we use the results of an asymptotic analysis of the equations of motion employed in [8, 9]. These papers obtained the result that for strong blowing the dividing streamline characterizing the zero value of the stream function is a straight line, and the region bounded by it has the shape of the wedge

$$y_0 = (\pi^2 M^2/2)^{1/3} x. \tag{1}$$

Therefore, to describe the velocity distribution in the boundary layer with uniform blowing and allowing for the induced pressure gradient, we use the similarity family of Falkner–Skan profiles appropriate to flow over a permeable wedge with semiopening angle

$$f''' + f''f + \beta(1 - f'^2) = 0,\ \eta = 0,\ f = -f_w,$$
$$f' = 0;\ \eta = \infty,\ f' = 1. \tag{2}$$

Thus, it is assumed that the influence of blowing on the external flow may be an effective method of replacing the original problem by an equivalent one: flow over a body whose profile is formed by the dividing streamline between the blown gas and the incident flow. Then, assuming, on the basis of Eq. (1), that the blowing creates the same pressure gradient as in flow over a wedge, i.e., that these two flows are similar, we obtain the following relation between the pressure parameter and the blowing intensity:

$$\beta = \frac{2}{\pi} \operatorname{arctg} \left(\frac{\pi^2}{2} M^2 \right)^{1/3}. \tag{3}$$

We used Eq. (3) to calculate the velocity distributions shown in Fig. 1. In a comparison with the experimental data of [5], obtained for a Reynolds number of $Re_x = 5 \cdot 10^3$, we find satisfactory agreement of the results for all intensities of transverse mass blowing.

Energy Institute im. G. M. Krzhizhanovskii, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 566-571, October, 1986. Original article submitted August 19, 1985.

0022-0841/86/5104-1167$12.50

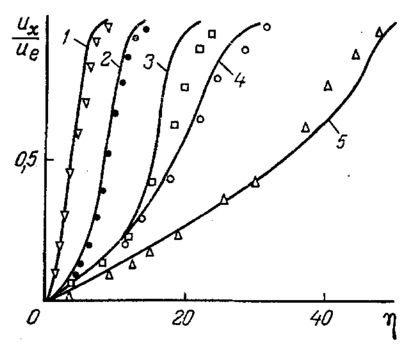

Fig. 1

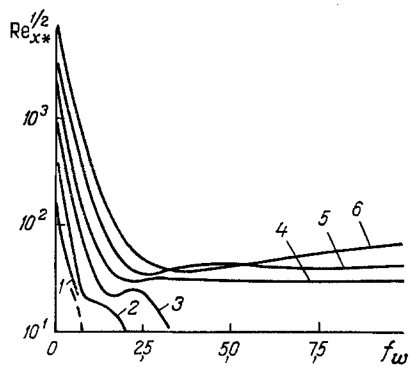

Fig. 2

Fig. 1. Velocity distribution across the boundary layer with uniform blowing: 1) M = 0.0058; 2) 0.013; 3) 0.032; 4) 0.05; 5) 0.11.

Fig. 2. Critical Reynolds number of the similarity boundary layer as a function of the blowing parameter: 1) β = 0.0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5.

On the basis of the similarity described above we can calculate the stability of the laminar boundary layer on a permeable flat plate with uniform blowing, for which one must calculate the stability characteristics of the Falkner–Skan similarity flows over a wide range of variation of the pressure and blowing intensity parameters.

2. The stability of laminar flow in the boundary layer described by Eq. (2) was analyzed in a linear formulation. In this case, the equation for the amplitude of small perturbations in similarity variables has the form

$$\begin{aligned}\varphi^{IV} - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha\sqrt{(2-\beta)\,\mathrm{Re}_x}\,[(f' - C)(\varphi'' - \alpha^2\varphi) - \\ - f'''\varphi] - (f + (\beta - 1)\eta f')(\varphi''' - \alpha^2\varphi') + ((2\beta - 1)f'' + (\beta - 1)\eta f''')\varphi',\end{aligned} \quad (4)$$

as the velocity scale we choose the quantity u_e, and as the length scale – $\sqrt{(2-\beta)/\mathrm{Re}_x}\,x$. The boundary conditions for Eq. (4) are given by the requirement that there are no velocity fluctuations at the wall and that they attenuate at infinity according to the exponential law

$$\eta = 0,\ \varphi = \varphi' = 0;\ \eta = \infty,\ \varphi'' - \alpha^2\varphi = 0,\ \varphi' + \alpha\varphi = 0. \quad (5)$$

The calculations were done for the case of neutral perturbations, i.e., we assumed C_i = 0. The eigenvalue problem of Eqs. (4) and (5) is solved by the differential marching method described in [10].

For zero-gradient flow over a flat plate (β = 0) the calculated results coincide with the corresponding data obtained in [11]. For this case there is a critical value of the blowing parameter f_{w*} = 0.876, which as we approach it becomes difficult to calculate the stability, since there is separation of the boundary layer characterized by unbounded increase of its thickness and by the velocity gradient at the wall going to zero. To describe the flow near the separation point ($f_w \to f_{w*}$) as was shown in [12], one can use the solution of the Lock problem for the mixing layer of parallel flows

$$f''' + ff'' = 0;\ f(-\infty) = -f_{w_*},\ f'(-\infty) = 0,\ f'(\infty) = 1. \quad (6)$$

Thus, the problem of determining the stability of the similarity boundary layer as the blowing parameter approaches the critical value reduces to solving the Orr–Sommerfeld equation (4) for the jet flow described by Eq. (6). In solving Eq. (4) at the boundaries of the region we impose the conditions of exponential attenuation of the perturbations (the second conditions of Eq. (5)), which are translated to the finite point η_0 defining the position of the dividing streamline. As was shown in [10], a variation of the boundaries of the jet flow region can appreciably affect the critical Reynolds number. This property of the stability characteristics is the basis for calculating the critical Reynolds number dependence Re_{x*} f(w), for which we use the asymptotic formula connecting the coordinate of the dividing streamline and the blowing parameter [12]

$$\eta_0 = \frac{1}{f_{w_*}}\left[\ln\gamma^k + 1.0367 + 0.2328\left[\frac{\ln^2\gamma^k}{\gamma^k} + \frac{\ln\gamma}{\gamma^k}\right] + \frac{0.2137}{\gamma^k} + \ldots\right],$$

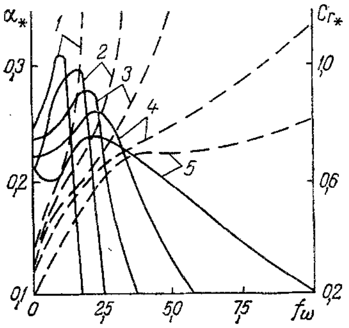

Fig. 3

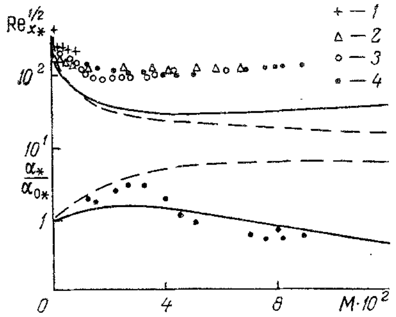

Fig. 4

Fig. 3. Critical wave number as a function of the blowing parameter: 1) β = 0.1; 2) 0.2; 3) 0.3; 4) 0.4; 5) 0.5.

Fig. 4. Influence of uniform blowing intensity on the critical stability parameter: 1) experiment [6]; 2) [14]; 3) [15]; 4) [16].

where k = 2.452, γ is determined from solution of the equation $k\varepsilon = \ln(\gamma^k)/\gamma^k$, $\varepsilon = |f_w - f_{w*}|$. The dependence of the critical Reynolds number, thus computed, on the blowing parameter (the broken line on Fig. 2) agrees well with the dependence obtained by solving Eqs. (4) and (5). It follows from the investigation conducted on the stability of the Lock flow of Eq. (6) that $Re_x \to 0$ for $f_w \to f_{w*}$, i.e., as we approach the separation point the laminar boundary layer becomes absolutely unstable.

For a negative pressure gradient in the external flow (β > 0) the equation of motion (2) has a solution for all values of β [1]. The entire range of variation of the pressure parameter can be divided into two regions characterizing the presence (0 < β < 0.5) or the absence (β > 0.5) of a knee point in the velocity profile with strong blowing. The stability characteristics for the case $\beta \geq 0.5$ were calculated in [13]. They obtained the result that as the blowing parameter increases the destabilizing influence of the transverse velocity component becomes stabilizing, which is determined by removal of velocity fluctuations in the external flow, and is confirmed by asymptotic analysis of the inviscid flow in the wall region. The graph of the dependence of the critical Reynolds number on the blowing parameter for values $0 \leq \beta \leq 0.5$ is shown in Fig. 2. It can be seen that even a slight S-shape of the velocity profile with strong blowing appreciably influences the critical Reynolds number, which either tends to a finite limit (for β = 0.4), or decreases (for β = 0.3), in contrast with the quadratic increase of $Re_{x*} \sim f_w^2$ for $\beta \geq 0.5$ [13]. For smaller values of β there is a sharp decrease of Re_{x*}, associated with the appearance of a sharply pronounced knee point in the unperturbed velocity profile. We note that in the range of variation of the pressure gradient parameter 0.1 < β < 0.5 the dependence $Re_{x*}(f_w)$ is characterized by the presence of a local minimum and maximum at moderate blowing.

Figure 3 shows the dependence $\alpha_*(f_w)$ (solid lines) and $C_{r*}(f_w)$ (broken lines). For small β the value α_* increases with increase of f_w, but for strong blowing it drops with increase of f_w for all values of β. Therefore, for strong blowing, as the blowing increases, perturbations arising in the laminar boundary layer during transition to the turbulent regime become longer in wavelength. This conclusion is confirmed also by the results of the experimental investigation of [5]. The critical velocity of propagation of the per turbations C_{r*} increases with increase of blowing for all β, but for β > 0.5 it tends to a finite limit determined by the asymptotic analysis of the stability of inviscid flow [13], and for $\beta \leq 0.5$ it increases without bound (the more sharply, the less is β). Evidently, the matter of unbounded increase of the velocity of propagation of perturbations arises from an error in using the boundary layer approximation with strong blowing. It is probable that in an analysis based on the full Navier–Stokes equations (within which the boundary conditions are valid which do not allow us to construct a continuous spectrum of eigenvalues in transition through $C_r = 1$ [10]) this effect would be absent.

3. Analysis of the stability of flow in the boundary layer with uniform strong blowing, using an exact solution of the equations of motion [3, 4], is impossible because of the presence of the critical blowing parameter, for which, as in the case of a Blasius profile on a

permeable flat plate, there is flow separation from the wall. The stability characteristics of this type of flow, using the approximate solution of [7], were analyzed in [6]. The results of a numerical solution [6] of the modified Orr–Sommerfeld equation, taking account of the transverse velocity component, are shown by broken lines in Fig. 4; Fig. 4 also shows the experimental data obtained in [6, 14-16]. The quantitative divergence between the calculated curve and the experimental data can be explained by the fact that the experimental points were determined from the point of appearance of regular perturbations of sinusoidal type of finite value, which corresponds to the start of transition of the laminar boundary layer to turbulent, while the linear theory investigates the stability of the flow to infinitely small perturbations, i.e., it determines the Reynolds number at loss of stability, which, as a rule, is less than the Reynolds number for the start of transition. However, the dependences $Re_{x*}(M)$ and $\bar{\alpha}_*(M)$ obtained for strong blowing prove not to be very satisfactory in a qualitative sense, since they predict a fall of Re_{x*} and an increase of $\bar{\alpha}_*$ with increase of the blowing intensity. This is due primarily to not accounting for the velocity distribution arising from the pressure gradient induced by blowing.

To determine the influence of the induced pressure gradient we use the results of calculating the stability characteristics of the similarity boundary layer on a permeable surface in the presence of a negative pressure gradient, as shown in Figs. 2 and 3. The results of the stability calculation for uniform blowing, obtained using Eq. (3), and of determining the blowing parameter $f_{w*} = M\sqrt{(2-\beta)Re_{x*}}$ are shown in Fig. 4 by solid lines. The nature of the dependence $Re_{x*}(M)$ obtained agrees with the experimental data [6, 14-16]. A comparison of the results obtained with the data of [15], where experiments were conducted in the entrance section of a two-dimensional channel, can be considered quite correct, since the negative pressure gradient due to flow constraint is less than that induced by the transverse mass flow. As was true for the experiments, there is an increase of flow stability with strong blowing as the blowing increases. The stabilizing influence of blowing is explained, according to Eq. (3), by an increase of the negative pressure gradient, under the action of which the S-shape is smoothed out (the knee point becomes less pronounced), and the fullness of the velocity profile increases. Figure 4 also shows the variation of the critical wave number, referenced to its value for M = 0, with increase of blowing intensity. The nature of the calculated dependence agrees with the experimental data of [15], where it was established that, as the transverse mass flow increases, the wave number initially grows, reaches a maximum (approximately in the intense blowing region), when its destabilizing influence becomes stabilizing, and then decreases. Thus, with strong blowing the boundary layer has longer-wave perturbations, which is the reason for the increase in the extent of the region of transition of the laminar flow regime to turbulent with increase of blowing. An analogous effect is confirmed also by the experimental data of [5, 15].

In conclusion, we note that, using the data obtained on the calculated stability of the Falkner–Skan problem with blowing, on the basis of the method of local similarity we can calculate the critical parameters for loss of stability of a wide class of flows on permeable surfaces characterized by the presence of a negative pressure gradient.

NOTATION

x, y, rectangular coordinates; u_e, velocity in the external flow; V_w, blowing velocity; ν, dynamic viscosity; $M = V_w/u_e$, blowing intensity; $F_w = M\sqrt{Re_x}$, $f_w = M\sqrt{(2-\beta)Re_x}$, blowing parameters; β, pressure gradient parameter; $Re_x = u_e x/\nu$, Reynolds number; $\eta = (y/x)\sqrt{Re_x(2-\beta)}$, similarity coordinate; f, similarity stream function; φ, amplitude of the velocity perturbations; α, $C = C_r + iC_i$, wave number and phase velocity of the distribution of perturbations; $\bar{\alpha}_* = \alpha_*/\alpha_0$. Subscripts: e, external flow; w, wall, *, critical.

LITERATURE CITED

1. O. A. Oleinik, Zh. Vyschisl. Mat. Mat. Fiz., 3, No. 3, 489-507 (1963).
2. A. I. Suslov, Prikl. Mat. Mekh., 38, No. 1, 166-169 (1974).
3. D. Catherall, K. Stewartson, and P. G. Williams, Proc. R. Soc. Lond., Ser. A, 284, No. 1398, 370-396 (1965).
4. N. A. Jaff, Proc. R. Soc. Lond., Ser. A, 317, No. 1530, 393-405 (1970).
5. A. A. Klimov, "Laminar and transitional flow regimes on porous and perforated surfaces with blowing of various gases," Candidate's Dissertation, Technical Sciences, Moscow (1978).
6. V. M. Eroshenko, L. I. Zaichik, A. A. Klimov, and A. V. Pershukov, Teplofiz. Vys. Temp., 22, No. 1, 83-86 (1984).

7. Inger, Raket. Tekh. Kosmn., No. 10, 196-197 (1966).
8. J. B. Klemp and A. Acrivos, J. Fluid Mech., 51, Pt. 2, 337-356 (1972).
9. F. T. Smith, J. Fluid Mech., 60, Pt. 2, 241-255 (1973).
10. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability and Turbulence [in Russian], Novosibirsk (1977).
11. T. S. Chen, E. M. Sparrow, and F. K. Tsou, J. Fluid Mech., 50, Pt. 4, 741-750 (1971).
12. D. R. Kassoy, SIAM J. Appl. Math., 18, No. 1, 29-40 (1970).
13. V. M. Eroshenko, L. I. Zaichik, and V. A. Pershukov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 60-64 (1984).
14. P. A. Libby, L. Kaufman, and R. P. Harrington, J. Aeron. Sci., 19, No. 2, 127-134 (1952).
15. A. L. Ermakov, V. M. Eroshenko, A. A. Klimov, et al., Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6, 114-123 (1972).
16. B. N. Baskarev, V. P. Motulevich, and Yu. N. Terent'ev, Inzh.-Fiz. Zh., 33, No. 2, 197-203 (1977).

FLOW OF A VISCOUS LIQUID FILM ON THE SURFACE OF A ROTATING DISK

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UDC 532.516

Results are presented from numerical calculations of steady-state axisymmetric flow of a film of viscous incompressible liquid over the surface of a plane rotating disk.

Film flow of a liquid over the surface of a rotating disk is found in many technological processes, the calculation of which requires knowledge of the hydrodynamic characteristics of such a flow. A number of theoretical and experimental studies have been dedicated to this question [1-5]. Dorfman [1] presented results of calculations by the difference method for the case of uniform initial velocity component profiles, [2-4] considered asymptotic solutions for relatively thin films, while [5] numerically determined a solution of special form. The present study will use the colocation method of [3], which allows calculations for a wide range of parameter values.

Let a viscous incompressible liquid be supplied near the axis of rotation of the disk at a constant volume flow rate Q. In analogy to [3], the velocity components u_r, u_θ, u_z in a fixed cylindrical coordinate system r, θ, z fixed to the center of rotation of the disk are represented in the form

$$u_r = \omega r \delta^2 u,\ u_\theta = \omega r(1+\delta^2 v),\ u_z = \omega H_0 \delta^2 w.$$

The quantity δ appearing in $\sqrt{\nu/\omega}$ is the thickness of the boundary layer which develops near an infinitely large disk rotating in an infinite liquid volume [6].

Without considering surface tension the system of equations and boundary conditions describing steady-state axisymmetric flow of the film, to the accuracy of terms of the order $(H_0/r)^2$, has the form [3]:

$$\frac{\partial u}{\partial x} + 2u + \frac{\partial w}{\partial y} = 0, \tag{1}$$

$$\frac{\partial^2 u}{\partial y^2} + 1 + 2\delta^2 v - \delta^4\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial y} + u^2 - v^2\right) = 0, \tag{2}$$

$$\frac{\partial^2 v}{\partial y^2} - 2\delta^2 u - \delta^4\left(u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial y} + 2uv\right) = 0, \tag{3}$$

M. V. Lomonosov Moscow State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 571-575, October, 1986. Original article submitted July 19, 1985.

0022-0841/86/5104-1171$12.50